Chapter 28

1

Direct Current Circuits

1

Lecture 01

2

- Electromotive Force
- Resistors Combination

2

Circuit Analysis

3

- Simple electric circuits may contain batteries, resistors, and capacitors in various combinations.
- For some circuits, analysis may consist of combining resistors.
- In more complex complicated circuits, Kirchhoff's Rules may be used for analysis.
  - These Rules are based on conservation of energy and conservation of electric charge for isolated systems.

3

Direct Current

4

- When the current in a circuit has a constant direction, the current is called **direct current.**
- Because the potential difference between the terminals of a battery is constant, the battery produces direct current.

4

Electromotive Force

5

- The battery is known as a source of electromotive force (emf) (ε).
- The emf of a battery is the maximum possible voltage the battery can provide between its terminals.
- The phrase electromotive force is an unfortunate historical term, describing not a force, but rather a potential difference in volts.
- The battery will normally be the source of energy in the circuit.
- The positive terminal of the battery is at a higher potential than the negative terminal.
- We consider the wires to have no resistance.

Internal Battery Resistance

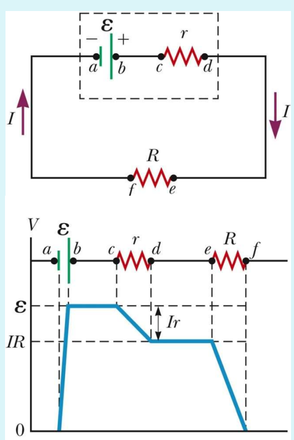
6

- If the internal resistance is zero, the terminal voltage equals the emf.
- In a real battery, there is internal resistance, r.
- The terminal voltage:

$$\Delta V = V_d - V_a$$

$$\Delta V = \varepsilon - Ir$$

- The emf is equivalent to the *open-circuit* voltage.
  - This is the terminal voltage when no current is in the circuit.
  - This is the voltage labeled on the battery.



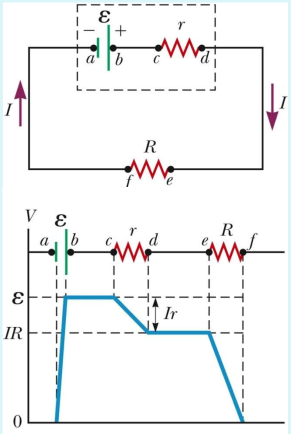
Load Resistance

7

- The actual potential difference between the terminals of the battery depends on the current in the circuit.
- The terminal voltage also equals the voltage across the external resistance.

$$\Delta V = V_e - V_f$$
$$\Delta V = IR$$

  - The load resistance is just the external resistor.
  - In general, the load resistance could be any electrical device.



7

Current:

8

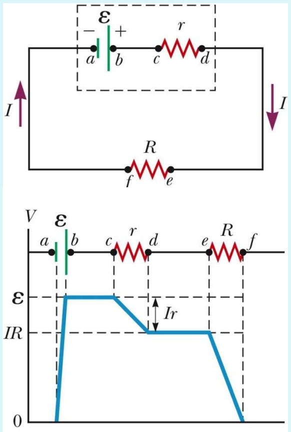
- The current in this simple circuit:

$$V_e - V_f = V_d - V_a$$
$$IR = \varepsilon - Ir$$
$$\boxed{I = \frac{\varepsilon}{R+r}}$$

- Note that:

$\Delta V = \varepsilon$

$$, \ IF \begin{cases} I = 0, \ open\ circuit \\ r = 0, \ NO\ internal\ resistance \end{cases}$$



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Power

9

- The total power output of the battery is

$$P = I\Delta V = I\varepsilon$$

- This power is delivered to the external resistor (I^2R) and to the internal resistor (I^2r).

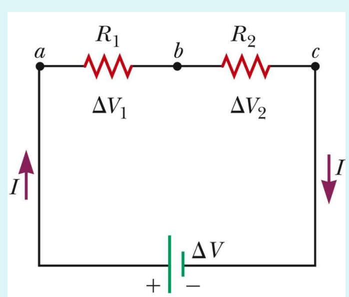
$$\boxed{I\varepsilon = I^2R + I^2r}$$

9

Resistors in Series

10

- When two or more resistors are connected end-to-end, they are said to be in series.
- For a series combination of resistors, the currents are the same in all the resistors because the amount of charge that passes through one resistor must also pass through the other resistors in the same time interval.
- The potential difference will divide among the resistors such that the sum of the potential differences across the resistors is equal to the total potential difference across the combination.



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Resistors in Series, cont

11

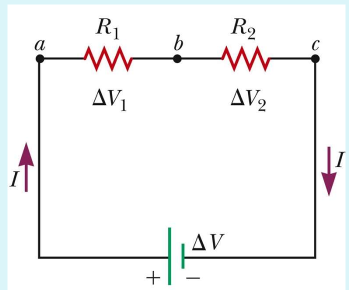
- Currents are the same

$$I = I_1 = I_2 = \cdots$$

- Potentials add

$$\Delta V = \Delta V_1 + \Delta V_2 + \cdots$$

  - Consequence of **Conservation of Energy**



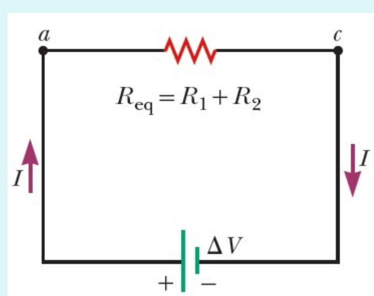
Equivalent Resistance – Series

12

- The equivalent resistance has the same effect on the circuit as the original combination of resistors.

$$R_{eq} = R_1 + R_2 + \cdots$$

- The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances.
- If one device in the series circuit creates an open circuit, all devices are inoperative.
- The equivalent **is always larger than** the largest resistor in the group.



Resistors in Parallel

13

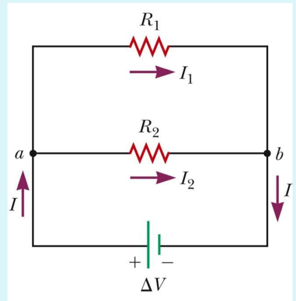
- The potential difference across each resistor is the same because each is connected directly across the battery terminals.

$$\Delta V = \Delta V_1 = \Delta V_2 = \cdots$$

- A **junction** is a point where the current can split.
- The current, I, that enters junction must be equal to the total current leaving that junction.

$$I = I_1 + I_2 + \cdots$$

  - The currents are generally not the same.
  - Consequence of **conservation of electric charge**



13

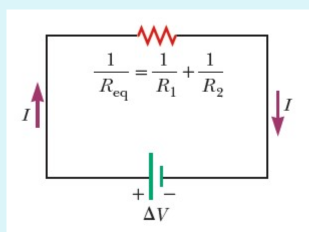
Equivalent Resistance – Parallel

14

- Equivalent Resistance.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

- The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistance.
  - The equivalent **is always less than** the smallest resistor in the group.



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Resistors in Parallel, Final

15

- In parallel, each device operates independently of the others so that if one is switched off, the others remain on.
- In parallel, all of the devices operate on the same voltage.
- The current takes all the paths.
  - The lower resistance will have higher currents.
  - Even very high resistances will have some currents.
- **Household circuits** are wired so that electrical devices are connected in **parallel**.

Terminal Voltage of a Battery-1

Friday, 29 January, 2021 21:25

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A battery has an emf of $\mathcal{E} = 12.0\,V$ and an internal resistance of $0.0500\,\Omega$. Its terminals are connected to a load resistance of $3.00\,\Omega$.

- Find the current in the circuit and the terminal voltage of the battery.
- Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

SOLUTION

- the current in the circuit:

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0V}{3.00\Omega + 0.0500\Omega} = 3.93A$$

the terminal voltage:

$$\Delta V = \mathcal{E} - Ir = 12.0V - (3.93A)(0.0500\Omega) = 11.8V$$

To check this result, calculate the voltage across the load resistance R:

$$\Delta V = IR = (3.93A)(3.00\Omega) = 11.8V$$

- the power delivered to the load resistor:

$$P_R = I^2 R = (3.93A)^2 (3.00\Omega) = 46.3W$$

the power delivered to the internal resistance:

$$P_r = I^2 r = (3.93A)^2 (0.0500\Omega) = 0.772W$$

the power delivered by the battery by adding these quantities:

$$P = P_R + P_r = 46.3W + 0.772W = 47.1W$$

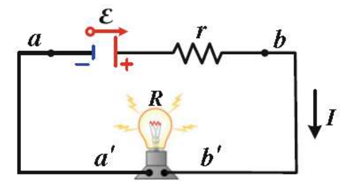
Terminal Voltage of a Battery-2

Friday, 29 January, 2021 21:25

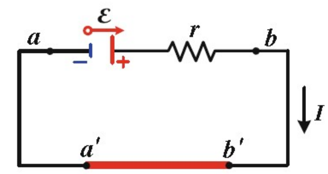
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A device is connected to a battery that has an emf $\mathcal{E} = 9V$ and internal resistance $r = 0.02\Omega$. Find the current in the circuit and the terminal voltage of the battery when the device is a:

- light bulb that has a resistance $R = 4\Omega$.
- conducting wire having zero resistance, i.e. the battery is short circuited by this conductor.



(a)



(b)

the value of the current is:

$$I = \frac{\mathcal{E}}{R + r} = \frac{9}{4 + 0.02} = 2.24A$$

the terminal voltage of the battery will be given by:

$$\Delta V = \varepsilon - Ir = 9 - 2.24 \times 0.02 = 8.96V$$

When we use a conducting wire, it is as if we have a device of $R = 0$. This results in a current and terminal voltage of the battery as follows:

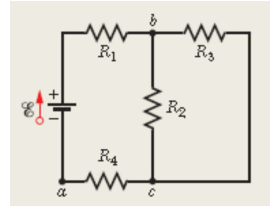
$$I = \frac{\mathcal{E}}{r} = \frac{9}{0.02} = 450A$$

$$\Delta V = \mathcal{E} - Ir = 9 - 450 \times 0.02 = 0V$$

Such large values for the current I would result in a very quick depletion of the battery as all of its stored energy would be quickly transferred to the conducting wire in the form of heat energy. The term “short circuit” is applied to such cases, and they can cause fire or burns.

Resistors in parallel and in series

Monday, 1 March, 2021 20:46



The figure shows a multiloop circuit containing one ideal battery and four resistances with the following values:

$R_1 = 20\ \Omega, R_2 = 20\ \Omega, R_3 = 30\ \Omega, R_4 = 8\ \Omega$ and $\mathcal{E} = 12\ V$

- What is the current through the battery?
- What is the current through R_2?
- What is the current through R_3?

Note carefully that R_1 and R_2 are *not* in series and thus cannot be replaced with an equivalent resistance. However, R_2 and R_3 are in parallel, so we can find their equivalent resistance R_{23},

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{(20\Omega)(30\Omega)}{50\Omega} = 12\Omega.$$

We can now redraw the circuit as in Fig. *c* ; note that the current through R_{23} must be i_1 because charge that moves through R_1 and R_4 must also move through R_{23}. For this simple one-loop circuit, the loop rule (applied clockwise from point a as in Fig. *d*) yields

$$+\varepsilon - i_1 R_1 - i_1 R_{23} - i_1 R_4 = 0.$$

Substituting the given data, we find

$$12V - i_1(20\Omega) - i_1(12\Omega) - i_1(8.0\Omega) = 0,$$

which gives us

$$i_1 = \frac{12V}{40\Omega} = 0.30A.$$

(b) What is the current i_2 through R_2?

We must now work backward from the equivalent circuit of Fig. *d*, where R_{23} has replaced R_2 and R_3.

Because R_2 and R_3 are in parallel, they both have the same potential difference across them as R_{23}.

We know that the current through R_{23} is $i_1 = 0.30A$. Thus, we can use $(R = V/i)$ and Fig. *e* to find the potential difference V_{23} across R_{23}. Setting $R_{23} = 12\Omega$ from (a),

$$V_{23} = i_1 R_{23} = (0.30A)(12\Omega) = 3.6V.$$

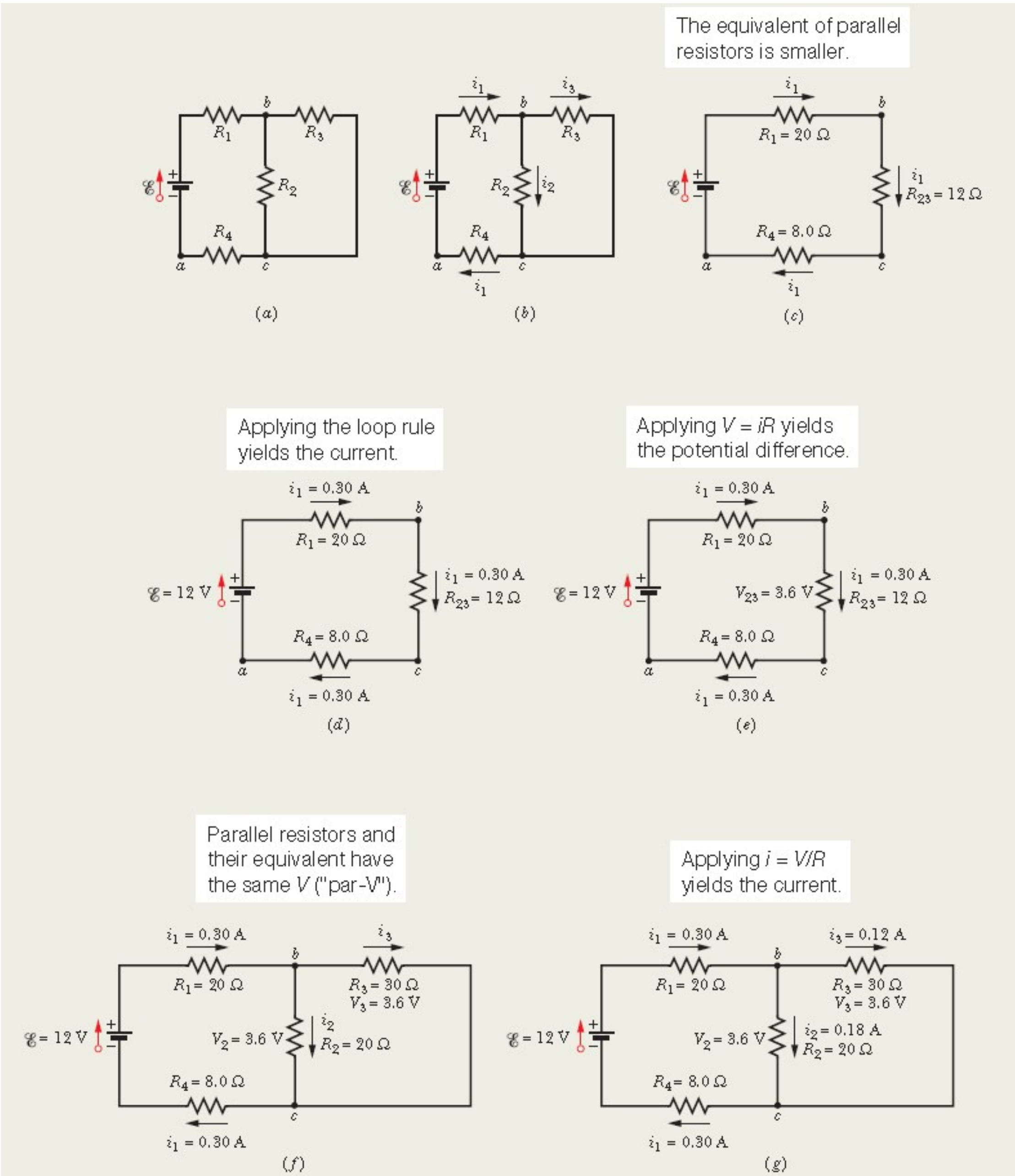
The potential difference across R_2 is thus also $3.6V$ (Fig. *f*), so the current i_2 in R_2 must be,

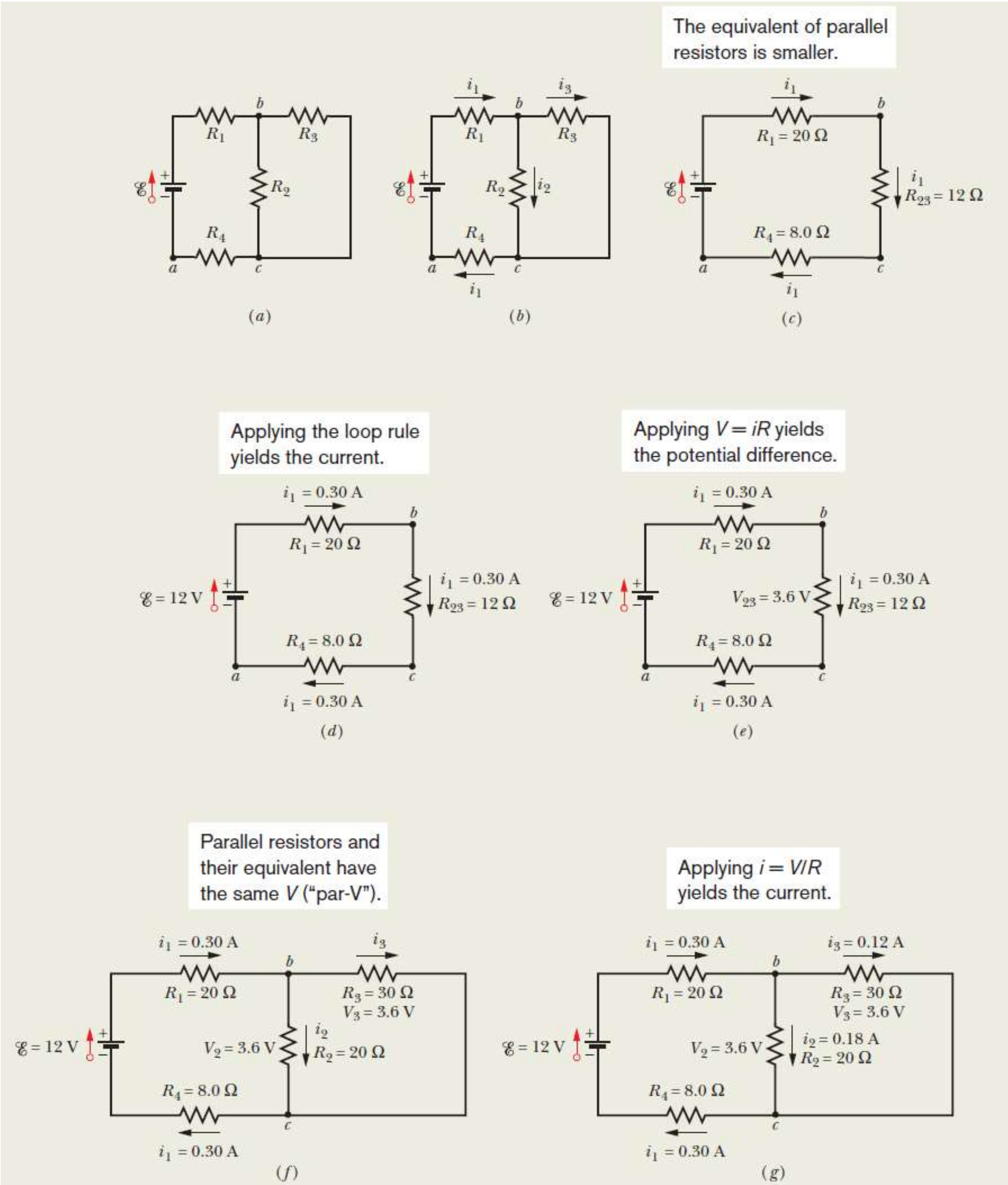
$$i_2 = \frac{V_2}{R_2} = \frac{3.6V}{20\Omega} = 0.18A.$$

(c) What is the current i_3 through R_3?

Rearranging this junction-rule result yields the result displayed in Fig. *g*:

$$i_3 = i_1 - i_2 = 0.30A - 0.18A = 0.12A.$$




The equivalent of parallel resistors is smaller.
Applying the loop rule yields the current.
Applying V = iR yields the potential difference.
Parallel resistors and their equivalent have the same V ("par-V").
Applying i = V/R yields the current.
a
b
c
R1
R2
R3
R4
i1
i2
i3
R1 = 20 Ω
R23 = 12 Ω
R4 = 8.0 Ω
ℰ = 12 V
i1 = 0.30 A
V23 = 3.6 V
R3 = 30 Ω
V3 = 3.6 V
V2 = 3.6 V
R2 = 20 Ω
i3 = 0.12 A
i2 = 0.18 A
(a)
(b)
(c)
(d)
(e)
(f)
(g)

Find the Equivalent Resistance

Tuesday, 2 February, 2021 21:25

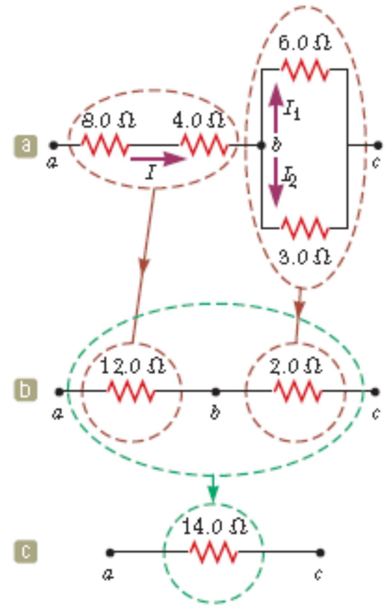
Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Four resistors are connected as shown.

- Find the equivalent resistance between points a and c.
- What is the current in each resistor if a potential difference of $42\,V$ is maintained between a and c?

SOLUTION



Imagine charges flowing into this combination from the left. All charges must pass through the first two resistors, but the charges split into two different paths when encountering the combination of the 6.0-Ω and the 3.0-Ω resistors.

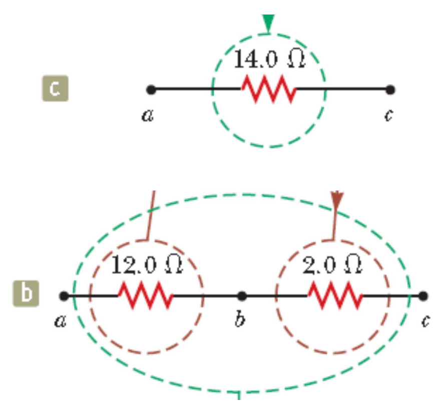
Find the equivalent resistance between a and b of the 8.0-Ω and 4.0-Ω resistors, which are in series (left-hand red-brown circles):

$$R_{eq} = 8.0\Omega + 4.0\Omega = 12.0\Omega$$

Find the equivalent resistance between b and c of the 6.0-Ω and 3.0-Ω resistors, which are in parallel (righthand red-brown circles):

$$\frac{1}{R_{eq}} = \frac{1}{6.0\Omega} + \frac{1}{3.0\Omega} = \frac{3}{6.0\Omega}$$

$$R_{eq} = \frac{6.0\Omega}{3} = 2.0\Omega$$

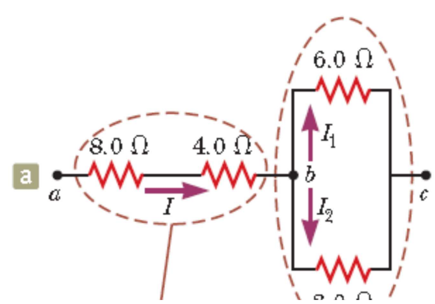


The circuit of equivalent resistances now looks like Figure b. The 12.0-Ω and 2.0-Ω resistors are in series (green circles). Find the equivalent resistance from a to c:

$$R_{eq} = 12.0\Omega + 2.0\Omega = 14.0\Omega$$

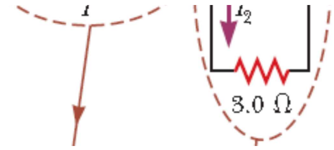
This resistance is that of the single equivalent resistor in Figure c.

SOLUTION



The currents in the 8.0-Ω and 4.0-Ω resistors are the same because they are in series. In addition, they carry the same current that would exist in the 14.0-Ω equivalent resistor subject to the 42-V potential difference.

Use $(R = \Delta V/I)$ and the result from part (A) to find the current in the 8.0-Ω and 4.0-Ω resistors:



Use $(R = \Delta V/I)$ and the result from part (A) to find the current in the 8.0-Ω and 4.0-Ω resistors:

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42V}{14.0\Omega} = 3.0A$$

Set the voltages across the resistors in parallel in Figure a equal to find a relationship between the currents:

$$\Delta V_1 = \Delta V_2 \rightarrow (6.0\Omega)I_1 = (3.0\Omega)I_2 \rightarrow I_2 = 2I_1$$

Use $I_1 + I_2 = 3.0A$to find I_1:

$$I_1 + I_2 = 3.0A \rightarrow I_1 + 2I_1 = 3.0A \rightarrow I_1 = 1.0A$$

Find I_2:

$$I_2 = 2I_1 = 2(1.0A) = 2.0A$$

As a final check of our results, note that

$\Delta V_{bc} = (6.0\Omega)I_1 = (3.0\Omega)I_2 = 6.0V$ and

$\Delta V_{ab} = (12.0\Omega)I = 36V$; therefore,

$\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} = 42V$, as it must.

Three Resistors in Parallel

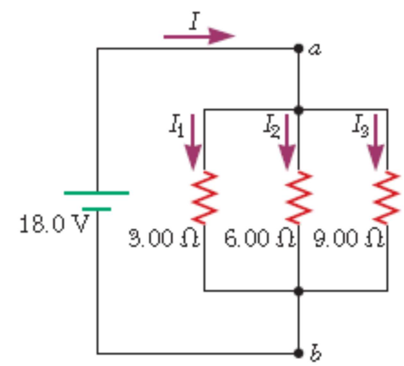
Tuesday, 2 February, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Three resistors are connected in parallel as shown. A potential difference of $18.0\,V$ is maintained between points a and b.

- Calculate the equivalent resistance of the circuit.
- Find the current in each resistor.
- Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.



SOLUTION

Notice that the current I splits into three currents I_1, I_2 and I_3 in the three resistors.

Find R_{eq}:

$$\frac{1}{R_{eq}} = \frac{1}{3.00\Omega} + \frac{1}{6.00\Omega} + \frac{1}{9.00\Omega} = \frac{11.0}{18.0\Omega}$$

$$R_{eq} = \frac{18.0\Omega}{11.0} = 1.64\Omega$$

SOLUTION

The potential difference across each resistor is 18.0 V. Apply the relationship $\Delta V = IR$ to find the currents:

$$I_1 = \frac{\Delta V}{R_1} = \frac{18.0V}{3.00\Omega} = 6.00A$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18.0V}{6.00\Omega} = 3.00A$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18.0V}{9.00\Omega} = 2.00A$$

SOLUTION

Apply the relationship $P = I^2R$ to each resistor using the currents calculated in part (B):

$$3.00 - \Omega: P_1 = {I_1}^2 R_1 = (6.00A)^2(3.00\Omega) = 108W$$

$$6.00 - \Omega: P_2 = {I_2}^2 R_2 = (3.00A)^2(6.00\Omega) = 54W$$

$$9.00 - \Omega: P_3 = {I_3}^2 R_3 = (2.00A)^2(9.00\Omega) = 36W$$

Part (C) shows that the smallest resistor receives the most power. Summing the three quantities gives a total power of 198 W. We could have calculated this final result from part (A) by considering the equivalent resistance as follows: $P = (\Delta V)^2/R_{eq} = (18.0V)^2/1.64\Omega = 198W$.